How to Measure the Diameters of Conical Chamfers

This is a short article on how to measure the major diameters of conically shaped chamfers. Most of the time, it is sufficient to know that a chamfer is large enough to ensure that a screw or rivet head will seat below the adjoining surface when installed. However, if a definite chamfer size is required, it is neither a straight forward matter to measure a chamfer or to create one. Making an accurate measurement of a chamfer diameter is not possible with a caliper or micrometer; a loupe type comparator can be used but is still not very accurate. Starrett, and maybe others, make chamfer gauges, but last I checked they were about $400 to buy, and if you needed to measure chamfers less than and greater than 90 degrees, you had to buy two. I have used these gauges, and they are rather finicky to use. If one is willing to do a little math, however, chamfers can be measured accurately. I know this because I had to calibrate the two Starrett gauges that I used. As a passing note, I have a large collection of machining books or reprints from the late 1880s through the 1960s, and it seems that when any but truly brief and / or elementary calculations are required, mathematical approaches are not mentioned. We live in the so called modern age of calculators, spreadsheets and other less useful things, so this should no longer be a problem.

If the included angle of the chamfer is known, the diameter may be accurately determined with the use of appropriately sized ball bearing. Using geometry and trigonometry, a formula can be derived so that the diameter can be calculated from measurements made with a caliper, micrometer or depth micrometer.

We will need a formula to help us measure existing chamfer diameters (*Formula (ia), Formula (ib)*) and another formula to create a chamfer diameter of a specific size (*Formula (ii)*):

$$D = 2\cdot\left[r + r/\sin\left(\frac{\theta}{2}\right) - h\right]\cdot\tan\left(\frac{\theta}{2}\right) \quad \text{for } h \leqslant r + r\cdot\sin\left(\frac{\theta}{2}\right) \qquad Formula(ia)$$

$$D = 2\cdot r\sin\left[\arccos\left(\frac{h-r}{r}\right)\right] \quad \text{for } 2\cdot r > h > r + r\cdot\sin\left(\frac{\theta}{2}\right) \qquad Formula(ib)$$

$$h = r + \frac{\mathrm{r}}{\left[\sin\left(\frac{\theta}{2}\right)\right]} - \frac{\mathrm{D}}{\left[2\cdot\tan\left(\frac{\theta}{2}\right)\right]} \quad \text{for } h \leqslant r + r\cdot\sin\left(\frac{\theta}{2}\right) \qquad Formula(ii)\ .$$

Now D is the diameter of the chamfer, it is the quantity we want to find in *Formula (ia)* and *Formula (ib)* and the size of the chamfer we want to create in *Formula (ii)*. The letter r is the ***radius*** of a ball bearing that we select that will ***fit inside the chamfer*** we are measuring or that we want to create. The Greek letter θ is the included angle of the chamfer we are measuring or want to make. The letter h represents the distance that the top of the ball bearing, when placed inside the chamfer, stands proud of the surface of the part. Note: if the top of the ball bearing is below the surface of the part, we treat that distance as a negative number. *Photo 1* and *Photo 2* show some details. The reason we have two formulas for the diameter (D) of the chamfer has to do with the diameter of the ball bearing. If the ball bearing diameter is bigger than its radius plus the sin of half the included angle then the ball bearing is sitting on the top of the chamfer *and NOT inside the chamfer*. The diameter of the chamfer may still be calculated (using *Formula (ib)*), when the ball sits on top of the chamfer -- clearly the value of h cannot be larger than the diameter of the ball bearing. *Formula ib* may be useful as it can be used to calculate the diameter of any hole where a ball bearing can be set on top of the hole. Note, however, that *Formula (ib)* may give inaccurate results if the edge of the hole is not crisp (damaged, deburred or contains burrs), or when the diameter of the ball is much larger than the hole.

Photo 1

Photo 2

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The tools for measuring chamfers are simple. They consist of calipers, micrometers, depth micrometers or height gauges, a couple of ball bearings and maybe a couple of parallels. *Photo 3*, shows some of these tools. The tools that are best suited to the task really depend on the size and configuration of the parts that are chamfered. To calculate the size of a chamfer we need to measure the diameter of the ball bearing used, to obtain the value of r, measure the value of h, and know the angle of the chamfer. See photos 4 through 8 for details on measuring the ball bearing diameter, the stock size and ball height to determine the value of h. The chamfer tool was a 90 degree single flute chamfer cutter.

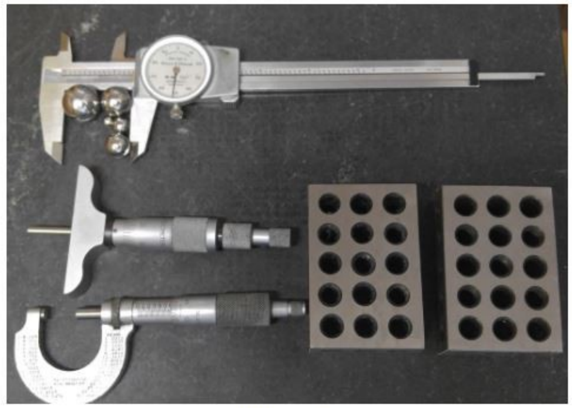

Photo 3

Using a micrometer only *Photo 4*, *Photo 5* and *Photo 6* give

$$r = 0.2498/2 = 0.1249$$

and

$$h = 0.6018 - 0.5000 = 0.1018$$

So we can calculate D as follows using *Formula (ia)*:

$$D = 2\cdot\left[0.1249 + 0.1249/\sin\left(\frac{90^o}{2}\right) - 0.1018\right]\cdot\tan\left(\frac{90^o}{2}\right)$$

or

$$D = 2\cdot[0.1249 + 0.1249/0.7071 - 0.1018]\cdot 1 = 0.3995 \quad .$$

Photo 4

Photo 5

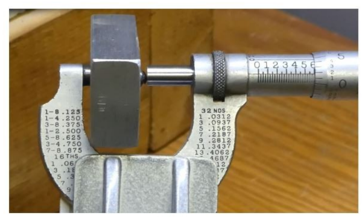

Photo 6

Notice that the caliper sanity check gives a diameter of something less than 0.416, which is over sized according to *Photo 9 (see below)*.

Using the depth micrometer as shown in *Photo 7* and *Photo 8* we have

$$h = 0.602 - 0.500 = 0.102$$

and Formula (i) yields

$$D = 0.3991 \quad .$$

Photo 7

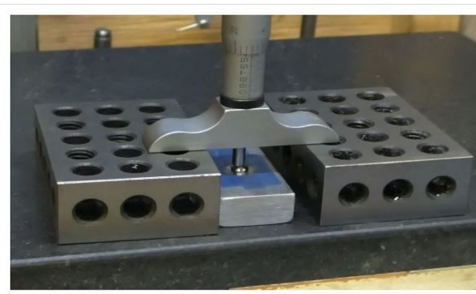

Photo 8

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Photo 9 is just a "sanity check", but shows how inaccurate a caliper measurement can be.

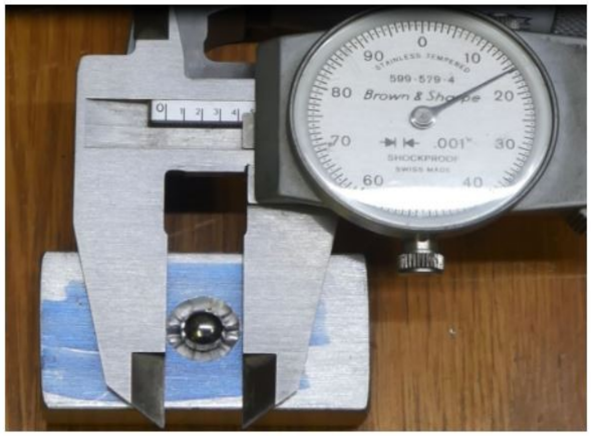

Photo 9

Formula (ii) is simply *Formula (ia)* solved for h, given a chosen value for D. The utility of this formula gives us a value of h for a given chamfer angle, chosen chamfer diameter and particular ball bearing radius. So, for example, if we wanted an 82.5 degree chamfer with a diameter of 0.5" and a 0.375" diameter ball bearing (think 1/4-20 flat head machine screw) we would calculate h as follows:

$$h = 0.1875 + \frac{0.1875}{\left[\sin\left(\frac{82.5^o}{2}\right)\right]} - \frac{0.5}{\left[2 \cdot \tan\left(\frac{82.5^o}{2}\right)\right]}$$

or

$$h = 0.1875 + \frac{0.1875}{[0.6593]} - \frac{0.5}{[2 \cdot 0.8770]} = 0.1868 \quad .$$

If we want to create the chamfer we just calculated h for, we would use the chamfering tool, **but create a shallow chamfer** and start to measure it by finding the value of h using our 0.375" diameter ball bearing. Suppose we found that h was 0.2012" for the shallow chamfer. By subtracting 0.1868" from 0.2012" we would obtain 0.0144". So to create our desired chamfer, we need to deepen our trial chamfer by 0.0144". If we are setup on a vertical mill, we just need to raise the table 0.014", or place a 0.014" shim under the part. If we are using a drill press where we can't control the table accurately, so the best method is to shim the part. As a practical matter, since 0.014" shims are hard to come by (even if you have gauge blocks), it is easier to place a gauge block or two under the part before cutting the trial chamfer, and then replacing the block with (a) taller one(s).

In an effort to make these calculations easier to deal with, here is a link to a web page of mine, that has a calculator with which you can calculate D as in *Formula (ia) Formula (ib),* as applicable, and h as in *Formula (ii)*. You only need to enter values for the ball **diameter**, the included angle and the measured value for h. Some pages of the website may be under construction, but the calculator page is active. It is suitable for both imperial and metric values with angles in degrees. The link is: https://thelocomachinist.tech/calculators/chamfers/.

Setting Blocks

If you make chamfers of different sizes on a regular basis, it will be helpful to make a Setting Block for each included angle that you regularly use. In my case, I regularly use chamfers with included angles of 60°, 82.5°, 90° and 100°. I have made a setting block for each of these angles. I have stamped on each one the angle, the ball diameter (B), the thickness of the block (T) and measured value of h (H). They are used in the following manner when I need to make a chamfer of accurate diameter. If I can hold the part in a vise, I place the Setting Block on

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parallels in the vise. To create the desired chamfer, the following values must be known: the Setting Block value of h (marked H on the block); the desired value of h; the thickness of the Setting Block (T), and the height of the surface to be chamfered above the parallels (the part height). Next, I set the quill stop on the drill press and raise the drill press table so the cutter touches the Setting Block. Following this, I replace the parallels with adjustable parallels so that when the cutter reaches the quill stop the chamfer will be cut to the correct depth. I calculate the height adjustment due to the difference between the Setting Block value of h and the desired value of h. Then I calculate the difference between the Setting Block height and the part height. To illustrate this, let us return to the example above. If the setting block value of h was 0.2012" I would calculate 0.2012"- 0.1868" = 0.0144". Then I would calculate 0.3750" - 0.1250" = 0.2500", where 0.3750 is the part height and 0.1250" is the Setting Block Height. To calculate the adjustable parallel height needed we recognize that the parallel needs to be increased by 0.0144" to make the chamfer deeper. We also recognize that we need to have the height of the parallel reduced by 0.2500" to set the part height at the Setting Block height. Thus, if we had the Setting Block on 1.0000" parallels we would want to set our adjustable parallels to 1.0000" + 0.0144" - 0.2500" = 0.7644". It is important to note that the change in h values and the change in (part / Setting Block) heights can be either positive or negative.

I have selected thicker materials and small chamfers for my Setting Blocks so that most of the time, the height of the parallels must be increased. If the items to be chamfered may be fastened to the mill table with an adjustable knee, the adjustable parallels will not be needed although the same calculations will be required. *Photo 10* shows the four Setting Blocks that I use. They can be made of steel, or aluminum if they are treated with care in use.

Photo 10

I would like to make a comment for those who do their own CNC programming. Establishing the vertical (and horizontal) offsets for chamfer cutters is a nuisance, in case you may not have noticed. You should be able to use these formulas and Setting Blocks to establish your cutter Z direction offsets. We have only dealt with conical chamfers here. In another article, I will tackle measuring linear or edge chamfers. As is commonly known, the recipe of "over (X or Y) then down (Z)" as prescribed by the tangent of one half the included angle, (i.e., $\tan(\theta/2)$ = X or Y/Z), can be used to extend chamfers upward or downward smoothly as needed. This is true of conical and linear chamfers.

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Derivation of the Formulas

Figure 1, below, is a sketch of the cross section of a chamfer with a ball bearing (of radius r) set inside of the existing chamfer.

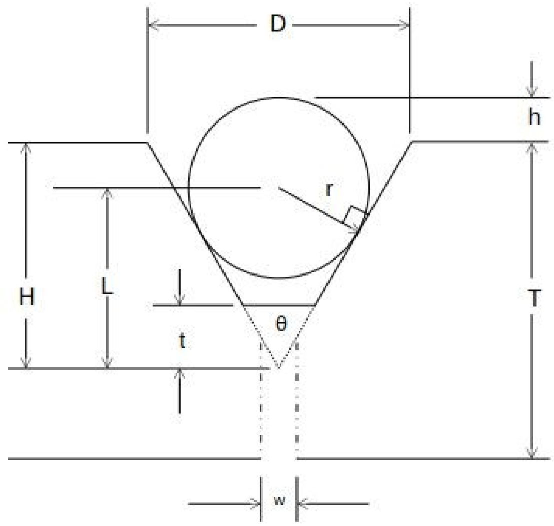

Figure 1

The significance of the various dimensions depicted in *Figure 1*,are listed below:

- D - the diameter of the chamfer at the surface of the part - this is what we want to find.
- r - the radius of the ball bearing. We will choose a ball bearing of small enough to fit inside the chamfer. It is convenient, but not necessary to select a ball whose top surface will extend above the top surface of the part, but it is not necessary. **It is important that the ball bearing diameter not be so large that it rests upon the edge of the conical hole.**
- θ - the included angle of the chamfer.
- h - this is the distance the ball bearing protrudes above above the surface of the part. If the top of the ball is above the surface it is a positive number; if the top of the ball is **below** the surface of the part it is a **negative** number. This is a measured quantity.
- T - this dimension is the thickness of the part. If this is a small number less than an inch or two (like bar or sheet) and easy to measure, knowing it may make the measurement process simpler. This is also a measured quantity.
- H - this is the distance from the theoretical "point" of the chamfer to the surface of the part. It may be calculated but it cannot be measured directly.
- L - this is the distance from the theoretical "point" of the chamfer to the center of the ball bearing. It may be calculated but it cannot be measured directly.
- t - this distance is the distance from the bottom of the chamfer as cut to the theoretical "point" of the chamfer. It may be possible to measure this and could be useful for CNC programming. See the note at the end of this article.
- w - this is the diameter of the hole for the screw or rivet. The size may be measured, but is not relevant for determining the chamfer size.

Here is the (brief) derivation of the formulas we need. Since the radius of the ball bearing is perpendicular to the wall of the chamfer we have

$$\frac{r}{L} = \sin\left(\frac{\theta}{2}\right)$$

so

$$L = r/\sin\left(\frac{\theta}{2}\right) \quad .$$

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Also we have

$$h+H \;=\; r+L$$

and

$$H \;=\; r+L-h \;\;=\;\; r+r/\sin\left(\frac{\theta}{2}\right)-h \;\;.$$

From basic trigonometry,

$$\tan\left(\frac{\theta}{2}\right) \;=\; \frac{D/2}{H}$$

so the diameter of the chamfer D is given by the formula

$$D \;=\; 2\cdot H\cdot\tan\left(\frac{\theta}{2}\right)$$

and substituting for H we can write

$$D \;=\; 2\cdot\left[r+r/\sin\left(\frac{\theta}{2}\right)-h\right]\cdot\tan\left(\frac{\theta}{2}\right) \qquad\qquad Formula(ia) \;\;.$$

This is a mouthful, but we can now calculate the chamfer diameter D, because we we know the included angle of the chamfer, θ, the radius of the ball bearing, r, and we can measure the distance from the top of the ball to the surface of the part. A simple example would be a 90 degree chamfer with a 0.25" diameter ball bearing that sticks up 0.1" above the surface of the part:

$$D \;=\; 2\cdot\left[0.125+0.125/\sin\left(\frac{90^o}{2}\right)-0.1\right]\cdot\tan\left(\frac{90^o}{2}\right)$$

so

$$D \;=\; 2\cdot\left[0.025+0.125/\sin(45^o)\right]\cdot\tan(45^o)$$

and finally

$$D=2\cdot[0.025+0.125/0.7071]\cdot(1)=2\cdot[0.025+0.1768]=0.4036$$

Now if we wanted to create a 90 degree chamfer of 0.5" diameter and we could solve F*ormula (ia)*, above for h. This would give us the formula below:

$$h \;=\; r+\frac{\mathrm{r}}{\left[\sin\left(\frac{\theta}{2}\right)\right]}-\frac{\mathrm{D}}{\left[2\cdot\tan\left(\frac{\theta}{2}\right)\right]} \qquad\qquad Formula(ii) \;\;.$$

For the above example, *Formula (ii)* tells us that h would have to be 0.0158 because

$$h \;=\; 0.125+\frac{0.125}{\left[\sin\left(\frac{90^o}{2}\right)\right]}-\frac{0.5}{\left[2\cdot\tan\left(\frac{90^o}{2}\right)\right]}$$

or

$$h \;=\; 0.125+\frac{0.125}{[0.7071]}-\frac{0.5}{[\;2\;]} \;=\; 0.0158 \;\;.$$

This tells us how to achieve a chamfer of desired size: First calculated the desired value of h that corresponds to the diameter of the chamfer we want. Then machine the chamfer shallow of the desired size, and measure the distance from the top of the ball bearing to the surface of the part - lets call this k. If we subtract k from the value of h we desire, then we know how much deeper we need to machine the chamfer to get the diameter we want.

Formula (ib) may be derived from *Figure 2,* below. Note that the ball of radius r is sitting on top of the hole rather than sitting in the hole. The angle ϕ has no definite relationship to the chamfer angle θ.

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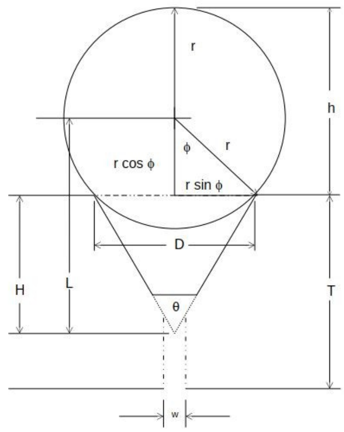

Figure 2

The derivation of *Formula (ib)* follows below. From *Figure 2*, we have that

$$h = r + r \cdot \cos(\phi)$$

so

$$\cos(\phi) = \frac{h-r}{r}$$

and

$$\phi = \operatorname{acos}\left(\frac{h-r}{r}\right) \quad .$$

Because

$$D = 2 \cdot r \cdot \sin(\phi)$$

we have

$$D = 2 \cdot r \cdot \sin\left[\operatorname{acos}\left(\frac{h-r}{r}\right)\right] \quad .$$

We note from *Figure 1*, that if

$$h > r + r \cdot \sin\left(\frac{\theta}{2}\right)$$

where θ is the included angle of the chamfer, the ball sits above the chamfer hole. Furthermore from *Figure 2*, it is obvious that the value of h cannot exceed the diameter of the ball, hence we have the constraints on *Formula(ib)*.

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